Turbulent Dispersion in Drag-Reducing Fluids

Turbulent pipe flow was examined by measuring the radial dispersion of dyed fluid introduced at an axial point source within the flow. From the classical theoretical results of G. I. Taylor it was possible thereby to calculate various parameters that characterize the structure of the turbulent flow field. A comparison was made between water, and aqueous solutions of polyethylene oxide (Polyox) at concentrations up to 50 ppm (by weight). Results indicate that Polyox affects the turbulence in the following ways: (1) intensity is reduced, (2) the energy spectrum is shifted toward low frequency, (3) dispersion occurs largely through large scale motion, and intermittency is increased. Indication is also seen that a simple exponential form for the Lagrangian correlation coefficient is superior to other often recommended models.

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SCOPE

While drag reduction phenomena in pipe flows are well documented, the mechanism whereby dilute polymeric additives affect turbulence is not completely understood. In particular, some previous work suggests that one must invoke specific interactions between the polymer and solvent in the neighborhood of the pipe wall to explain observations. The goal of this work was to examine turbulent phenomena in the axial core of a pipe flow, removed from wall influences.

A classical dye-tracer study was carried out by measuring the radial dispersion of dyed fluid introduced at an axial point source within the pipe. From the classical work of G. I. Taylor on dispersion by continuous movements it was possible to calculate the various parameters that characterize the structure of a turbulent flow field. A comparison was made between water, and aqueous solutions of polyethylene oxide (Polyox) at concentrations up to 50 ppm (by weight). Polyox is a well-known drag reducing additive.

CONCLUSIONS AND SIGNIFICANCE

Through the experiments described it was possible to obtain reliable measurements of turbulent parameters. The results indicate the following conclusions can be drawn:

1. Turbulence in a drag-reducing fluid has a smaller intensity than in the solvent alone, under otherwise identical conditions.
2. The energy spectrum is shifted, by the polymer additive, toward low frequency, large-scale, fluctuations.
3. "Fine structure" disappears in a drag-reducing fluid; dispersion occurs principally through large-scale motion, and intermittency is increased.
4. A simple exponential form for the Lagrangian correlation coefficient seems to be superior to other suggested models.

Of particular significance is the demonstration that the effect of drag-reducing fluids is not localized at the pipe wall but extends into the turbulent core. Thus, proposed mechanisms of drag reduction which rely entirely on alleged wall adsorption or orientation of the polymer molecules are incomplete.

It is well established that certain types of transport processes in fluids can be drastically altered by the addition of extremely small quantities of soluble polymeric substances to the fluids. The most common example of this phenomenon arises in the turbulent flow of liquids through pipes, where it is observed that the frictional drag at the pipe wall can be substantially reduced. Consequently, it is possible, using polymeric additives to increase the throughput at constant pressure or reduce the pressure drop required to achieve a given flow rate. The term *drag reduction* is now generally accepted as a nominal description of the process, and the term *drag-reducing fluid* is suggested for a polymer solution which exhibits drag reduction.

While a great deal of effort has been expended in the study of drag reduction phenomena, no universally accepted mechanistic explanation exists. Indeed there is still some controversy as to whether drag reduction phenomena should be approached from the viewpoint of continuum mechanics, or explicitly in terms of supposed interactions of individual polymer molecules with either the solvent, or with the rigid (pipe wall) boundary. The extensive background of both phenomenological observation and mechanistic interpretation will not be reviewed here in any detail. The reader is referred instead to the excellent review of Hoyt (1972) which cites more than two hundred references. Taylor (1973) also presents an extensive review of previous work, in the dissertation from

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which this paper is abstracted.

It is pertinent, as regards the motivation for the work being reported herein, to comment on one specific aspect of proposed mechanisms of drag reduction. This is with regard to the question of whether polymeric additives alter, in some way, the structure of turbulence throughout the entire flow field, either through continuum or noncontinuum effects, or whether the more localized interaction of polymeric additives with the bounding surfaces of the flow field provides the principal mode of flow alteration. Since drag reduction studies involve a solid surface against which drag is exerted, it is difficult to suggest the absence of boundary effects in such studies. What is needed, then, is a study of turbulence phenomena which do not involve, per se, a solid boundary. For this reason, the decision was made to examine the turbulent dispersion of a dye tracer in the core of a fully developed turbulent flow. The hope was that if one could demonstrate an effect of drag-reducing fluids on tracer dispersion, it might be possible to exclude those theories which rely exclusively on specific interactions of the polymeric additive localized at the pipe wall surface.

The view that drag reduction is a continuum phenomenon associated with elasticity of dilute polymer solutions is put forth in various forms by Seyer and Metzner (1967), Rodriguez et al. (1966), Hershey and Zakin (1967), Denn and Porteus (1971), and Astarita (1965). They propose that drag reduction occurs when a time scale of the turbulent fluctuations is of the same order of magnitude as the relaxation time of the solution. Gordon (1970) has discussed the question of appropriate time scales descriptive of the turbulent flow.

Specific wall interactions, and their relevance to drag reduction phenomena, are put forth by El'perin et al. (1963) who suggested the occurrence of adsorption of the polymer at the pipe wall. Arunachalam and Fulford (1971) have done experiments to measure the kinetics of adsorption and have found higher polymer concentrations near the wall in a pipe flow. Hand and Williams (1973) have proposed that an absorbed and entangled polymer layer must be involved in order to provide the dimensions necessary to reach into the outer wall region where most turbulence is produced and dissipated.

It is clear, however, that polymer absorption may not be a necessary condition for drag reduction since Gadd (1965) and Serth and Kiser (1970) found pronounced effects of dilute polymer solutions on the flow of free turbulent jets. Further, it should be noted that the continuum and absorption mechanisms are not necessarily exclusive, and it is possible that both factors are present to varying degrees in different turbulent flow configurations.

The purpose of this work, then, was to examine turbulent flow by means of measurement of the radial dispersion of a dye stream injected at a point along the axis of a pipe flow. As shown by G. I. Taylor (1935), considerable information can be extracted from careful measurements of the extent of dispersion. It was desired to measure the effect of polymeric additives on turbulence parameters such as the various length or time scales, the turbulent intensity, and the dissipation rate. Standard measuring techniques using hot wire or hot film probes were avoided because of the anomalous behavior observed by Smith et al. (1967) with these devices in dilute viscoelastic fluids. It was expected that a correlation between turbulent dispersion and drag reduction might shed some light on the mechanism whereby dilute polymeric additives affect turbulence.

THEORY

The statistical description of dispersion of tracer from a point source in a turbulent flow was first given by G. I. Taylor (1921). The fundamental result relates the mean squared displacement $\overline{y^2}$ to the mean squared fluctuating velocity $\overline{u^2}$ in the direction transverse to the mean flow, and to the Lagrangian velocity correlation coefficient $R_L(\tau)$:

$$\overline{y^2} = 2\,\overline{u^2} \int_0^t \int_0^{t} R_L(\tau)\, d\tau\, dt' \tag{1}$$

The correlation coefficient is defined as

$$R_L(\tau) = \frac{\overline{u(t)\, u(t-\tau)}}{\overline{u^2(t)}} \tag{2}$$

For small times τ it can be seen that $R_L(\tau)$ approaches unity, from which it follows that

$$\lim_{t\to 0} \overline{y^2} = \overline{u^2}\, t^2 \tag{3}$$

For long times the velocity fluctuations become uncorrelated, and it is possible to define a Lagrangian integral time scale as

$$T_L \equiv \int_0^\infty R_L(\tau)\, d\tau \tag{4}$$

Using Equation (4) it is possible to show that

$$\lim_{t\to\infty} \overline{y^2} = 2\,\overline{u^2}\, T_L\, t \tag{5}$$

By analogy to Einstein's (1905) formulation of diffusion by Brownian fluctuations, Taylor defined a turbulent dispersion coefficient E_D as

$$E_D \equiv \frac{1}{2} \frac{d\overline{y^2}}{dt} \bigg|_{t\to\infty} = T_L\, \overline{u^2} \tag{6}$$

From Equations (3) and (6) it is apparent that upon examining mean square displacement of a tracer at very short and very long times, it is possible to determine both $\overline{u^2}$ and T_L.

Many of the proposed mechanisms of drag reduction depend on an understanding of the dynamics of the small-scale structure of the turbulent field. An important quantity dependent on the small-scale structure is the viscous dissipation of energy. For isotropic turbulence in a newtonian fluid G. I. Taylor (1935) has found the rate of viscous dissipation to be

$$W = 15\, \mu\, \overline{u^2}/\lambda^2 \tag{7}$$

where μ is the shear viscosity and λ is the Taylor microscale.

An estimate of the Taylor microscale can be made, following the work of Brodkey (1967), and Beek and Miller (1959), with the result

$$\lambda^2 \cong 24\, \nu\, T_L \tag{8}$$

where ν is the kinematic viscosity. Hence any effect of polymer additives on small-scale structure can be determined by calculating T_L from dispersion measurements, and using Equation (8). Once λ is known, Equation (7) allows calculation of the rate of energy dissipation.

Another means of comparing two turbulent flows is in terms of the Lagrangian energy spectrum function $E_L(n)$, defined as the Fourier transform of $R_L(\tau)$:

$$E_L(n) = 4\,\overline{u^2} \int_0^\infty R_L(\tau) \cos 2\pi n \tau\, d\tau \tag{9}$$

If dispersion data are available in the form of $\overline{y^2}(t)$, then from Equation (1) it should be possible to determine the correlation function $R_L(\tau)$. Using Equation (9) one could then examine the dependence of the energy spectrum function on frequency n.

Thus it is apparent that if sufficiently accurate data on dispersion can be generated, it is possible to examine a variety of parameters characteristic of the structure of the turbulent field.

EXPERIMENT

A schematic drawing of the experimental system is shown in Figure 1. A large constant head reservoir feeds a long 2-in. transparent acrylic pipe which descends vertically to the floor below. Dye is added to fluid otherwise identical to that in the reservoir, and the dyed stream is injected isokinetically through a small capillary aligned along the pipe axis. In this way a point source of tracer is approximated. A sample of fluid is withdrawn isokinetically downstream by a suction tube which can be positioned with a micrometer. The sample is transferred to the cell of a photrometer whereby dye concentration is determined via light transmission.

The injection of dye was through a stainless steel tube of 0.063-in. O.D. and 0.050-in. I.D. The withdrawal of sample was through a stainless steel tube of 0.042-in. O.D. and 0.027-in. I.D. A more detailed description of the design and operation of the injection and withdrawal systems is given by Taylor (1973).

Friction factors were determined for each run by simultaneous measurement of pressure drop and flow rate through the test section. A Gilmont micrometric manometer was connected across a pair of taps three feet apart along the test section.

The polymer used to produce drag reduction was a high molecular weight polyethylene oxide (Polyox WSR-301, Union Carbide Corp.). Paterson and Abernathy (1970) have given the weight average molecular weight of WSR-301 as 8×10^6. Because of molecular weight variations among different batches it is advisable to characterize samples through viscometry. Relative viscosity was measured with an Ostwald viscometer and found to be 1.071 at 20°C and 50 wppm (parts per million by weight) polymer. Intrinsic viscosity was found to be 14.2 dl/g at 20°C. Using data of Shin (1965) this corresponds to a weight average molecular weight of 4×10^6.

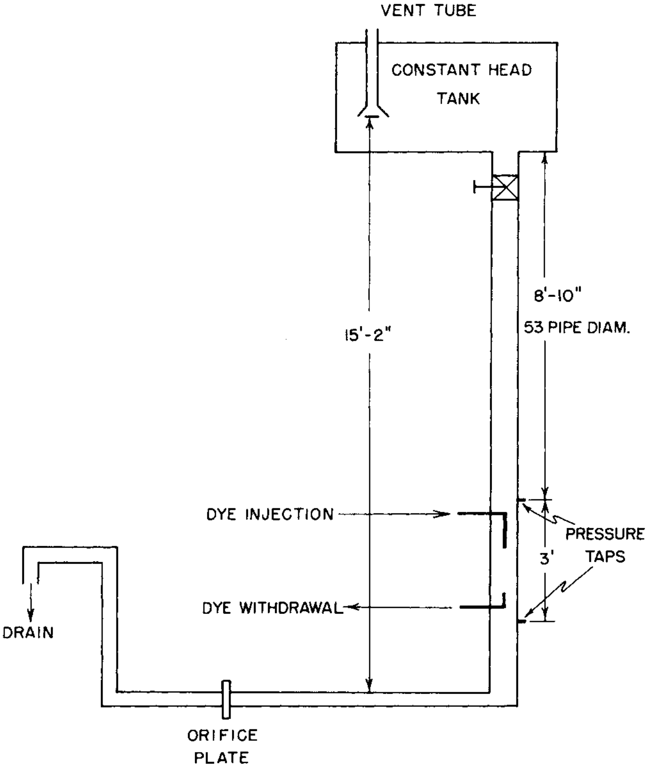

Fig. 1. Schematic of experimental system.

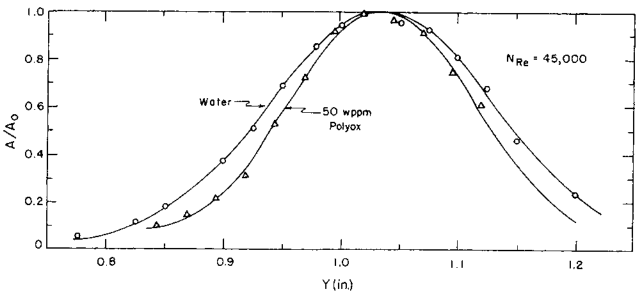

Fig. 2. Comparison of absorbance profiles across a tracer plume in water and 50 wppm Polyox.

The dye used was methyl violet, which shows very strong absorption at 580 mμ. Measurements of absorbance as a function of dye concentration were made with a Leitz Model M Photrometer at 580 mμ. The Beer-Lambert Law expressing linearity between dye concentration and absorbance was obeyed. Absorbance was found to be the same in distilled water, tap water, and tap water containing 100 wppm Polyox.

The concentration of dye in the injected stream was varied depending on the axial distance between injector and probe for a given experimental run. Between the minimum and maximum distances of 5.2 cm and 33 cm, the dye concentration in the injector reservoir varied from 100 to 500 wppm.

A detailed discussion of experimental procedure is given by Taylor (1973).

METHOD OF DATA ANALYSIS

Figure 2 shows data of absorbance of samples removed from different radial positions at a fixed axial point downstream of dye injection. The absorbance was normalized to the maximum (centerline) value, and since the Beer-Lambert Law was followed it was not necessary to convert these data to concentration values.

These curves were fitted with a Gaussian distribution of the form

$$A/A_0 = \exp\left[-\,(Y-\overline{Y})^2/2\overline{Y^2}\right] \qquad (10)$$

by selecting the best values of the mean $\overline{Y}$ and variance $\overline{Y^2}$ of the distribution. Graphical methods, as well as a linear least squares fit of Equation (10) (transformed to $\ln A/A_0$) failed to give sufficiently accurate values for $\overline{Y}$ and $\overline{Y^2}$. The method finally used was a nonlinear least squares routine known as Gaushaus, and described by Meeter (1965). It is a combination of the Gauss method with the method of steepest descent.

For a given run, at fixed pipe Reynolds number and Polyox concentration, $\overline{Y^2}$ was determined as a function of axial position x downstream of the dye injector. Distance was converted to time using $t = x/U_c$, and a plot of $\overline{Y^2}$ versus t was thereby constructed, such as in Figure 3. The variance $\overline{Y^2}$ was then interpreted to be the same as the mean squared displacement $\overline{y^2}$ of Equation (1).

Because of scatter in these data, it proved difficult to obtain values of $\overline{u^2}$ and E_D (or T_L) by direct application of Equations (3) and (6). An alternate procedure was used which involved fitting a model for the Lagrangian correlation coefficient $R_L(\tau)$ to the $\overline{Y^2}$ versus t data, using Gaushaus and Equation (1).

Frenkiel (19) has proposed several simple forms for $R_L(\tau)$ which have semi-empirical bases:

$$R_L(\tau) = \exp\,(-\,\tau/T_L) \qquad (11)$$

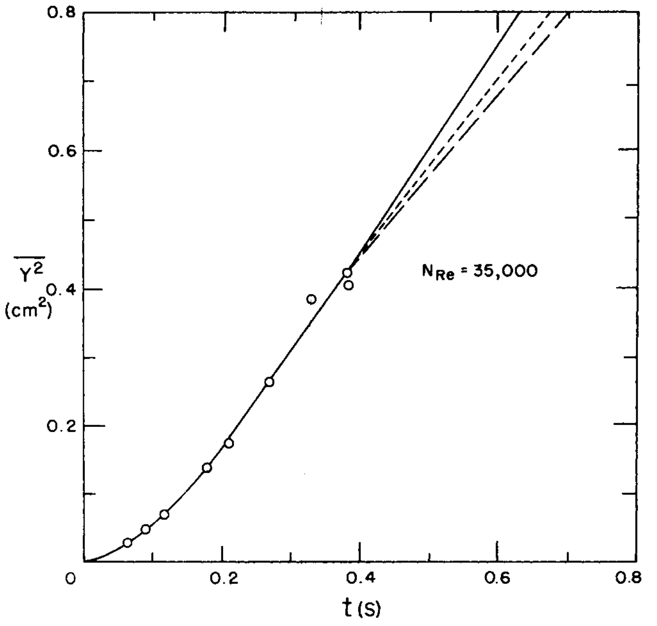

Fig. 3. Mean squared displacement vs. time for a tracer plume in water. Curves give best fits to the data for each of three assumed correlation functions:

$$R(\tau) = \begin{cases} \exp(-\tau/T_L) & \text{———} \\ \exp(-\pi\tau^2/4T_L^2) & \text{— —} \\ \exp(-\tau/2T_L)\cos(\tau/2T_L) & \text{- - - -} \end{cases}$$

$$R_L(\tau) = \exp(-\pi\tau^2/4\,T_L^2) \qquad (12)$$

$$R_L(\tau) = \exp(-\tau/2\,T_L)\cos(\tau/2\,T_L) \qquad (13)$$

Equation (1), then, with any of Equations (11), (12), or (13), provided a two-parameter model with which the $\overline{Y^2}$ versus t data could be fit.

RESULTS

Drag Reduction

To confirm that fully developed turbulent flow existed in the test section a series of pressure drop-flow rate runs were made at various Reynolds numbers using water containing no additives. Figure 4 shows data, presented as a friction factor plot, which are in good agreement with the Blasius relation given by Bird et al. (1960):

$$f = 0.079\,N_{Re}^{-1/4} \qquad (14)$$

In the course of dispersion measurements in drag reducing fluids pressure drop was routinely measured. Figure 4 shows results for 50 wppm Polyox at the four Reynolds numbers studied. The four points represent, collectively, an average of 36 individual measurements. The curve shown through the data is traced from the results of Goren and Norbury (1967) for 50 wppm Polyox in a 2-in. pipe.

The good agreement with both the classical Blasius relation as well as with the related study of Goren and Norbury suggests that fully developed turbulent flow exists in the test section and that accurate flow measurements have been made with our equipment.

Dispersion

Concentration (or absorbance) profiles were obtained and plotted as described earlier. The addition of Polyox produced significant changes in the character of the dispersion. Figure 2 shows a comparison of two runs for water, and 50 wppm Polyox, under otherwise identical conditions. Clearly, the dye plume in Polyox is narrower than in water. As described earlier, these concentration profiles could be fitted to a Gaussian distribution by a nonlinear least squares routine, yielding values for $\overline{Y^2}$. Then plots of $\overline{Y^2}$ versus t (such as in Figure 3) were fitted with another nonlinear least squares routine utilizing Equation (1) and an assumed form for $R_L(\tau)$, to yield values of T_L. Other turbulence parameters were then calculated as outlined above.

Figure 3 is typical of all the data treated in this study. It is difficult to distinguish by eye among the three correlation functions [Equations (11) to (13)] used to fit the data. It was not possible to obtain data at long times where the models are distinguishable. The error variance of each model was calculated from

$$\sigma^2 = \sum_{i}^{N} [\overline{Y^2}_{\text{exp}} - \overline{Y^2}_{\text{model}}]_i^2/(N-P) \qquad (15)$$

where N is the number of data points and P is the number of parameters. In this work, $P = 2$, the parameters being $\overline{u^2}$ and T_L.

Table 1 shows the error variances for each model, at four different Reynolds numbers, for water. It does not seem possible to decide on the best model on this basis. A similar conclusion is reached in examining the variances for the Polyox dispersion data.

Turbulence parameters obtained through this curve fitting procedure do, however, differ depending on which model for $R_L(\tau)$ is used. It was decided to recast $\overline{u^2}$ and T_L into the turbulent intensity (radial)

$$u' = \sqrt{\overline{u^2}}/U_c \qquad (16)$$

and the dispersion coefficient [Equation (6)]

$$E_D = \overline{u^2}\,T_L$$

Table 1. Error Variance from Fitting $R_L(\tau)$ to $\overline{Y^2}$ vs. t Water Data

	$10^4\,\sigma^2$		
N_{Re}	Equation (11)	Equation (12)	Equation (13)
17,800	6.4	5.6	5.8
24,500	1.5	2.7	1.9
35,000	2.2	2.5	2.3
45,000	2.4	3.0	2.6

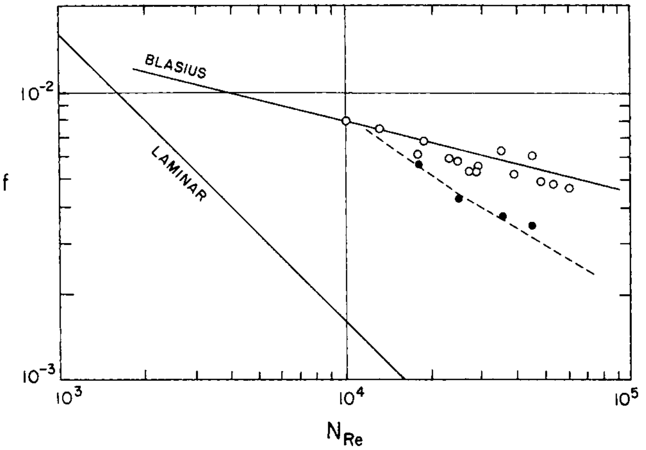

Fig. 4. Data for friction factor for flow in test section: water (○) and 50 wppm Polyox (●).

Figure 5 shows u' as a function of Reynolds number for the water data, using Equations (11) to (13) for $R_L(\tau)$. The curve is the best fit of data obtained by Goldstein et al. (1969) in experiments in a half-inch pipe flow of water, using a laser-velocimeter to measure longitudinal intensity. Since the pipe core can be expected to be nearly isotropic (Hinze, 1959), their longitudinal intensities should be similar to our radial intensities. These results suggest that Equation (12) is poorer than Equations (11) or (13).

Figure 5 shows calculated values of E_D from our water data, compared to similar data of Sheriff and O'Kane (1971) and Groenhof (1970). Equation (11) seems slightly superior to Equations (12) and (13) in this case. It appears then that the dispersion data obtained in water, when treated according to the methods outlined above, yields results for u' and E_D in good agreement with those of other investigators. On the basis of the small amount of data produced in our study, Equation (11) seems to provide a useful empirical relationship for $R_L(\tau)$.

Turning now to turbulence parameters extracted from dispersion measurements in drag-reducing fluids, one can examine the radial intensity u' in 50 wppm Polyox, shown as a function of Reynolds number in Figure 6. Equation (11) was used in obtaining u'. It would appear that the turbulent intensity is significantly reduced by the Polyox.

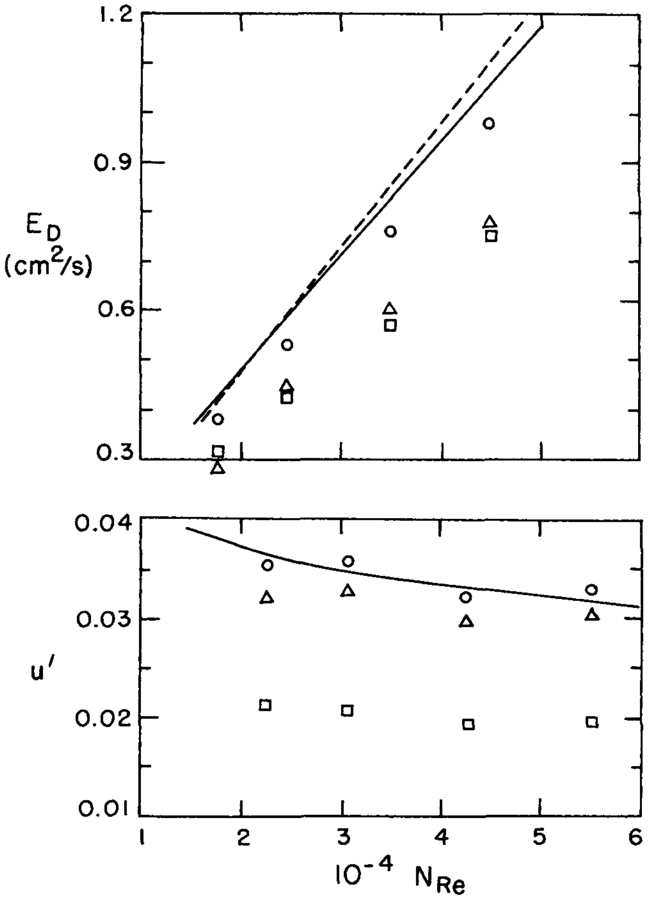

Fig. 5. Calculated radial intensity u' in water, using

$$R(\tau) = \begin{cases} \exp(-\tau/T_L) & \bigcirc \\ \exp(-\pi\tau^2/4\,T_L^2) & \square \\ \exp(-\tau/2\,T_L)\cos(\tau/2\,T_L) & \triangle \end{cases}$$

Solid line fits data of Goldstein et al. (1969). Calculated dispersion coefficient E_D in water. Symbols same as above. Lines are fits of data of Groenhof (1970) ———, and of Sheriff and O'Kane (1971) - - - - -.

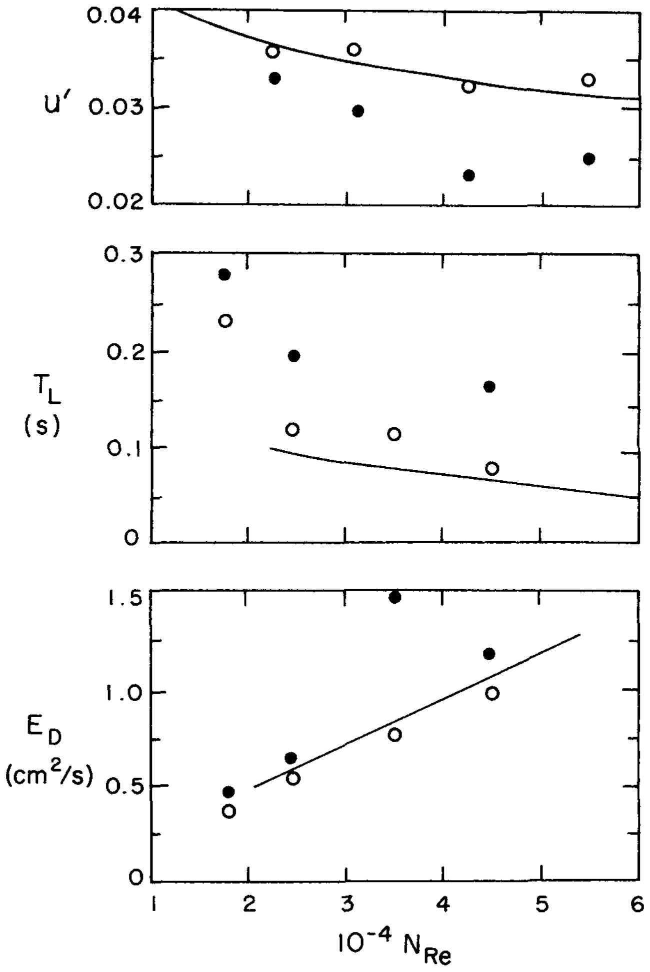

Fig. 6. Comparison of radial turbulence intensity u', Lagrangian integral scale T_L, and dispersion coefficient E_D in water, ○, and 50 wppm Polyox ●. Solid line fits water data of Groenhof (1970).

In Figure 6 the integral scale is calculated, using Equation (11), and water and 50 wppm Polyox results are compared. It would appear that T_L is significantly increased by the presence of Polyox. (Similar increases were observed using Equations (12) and (13), but these results have been omitted from display.)

There is quite a bit of scatter in the Polyox measurements; data at $N_{Re} = 35{,}000$ and 50 wppm are especially poor. It was observed that the dye plume tended to be considerably more intermittent in Polyox, presumably because of damping of the small scale motion. Consequently, concentration measurements at a given point, which in some sense are averaged over time, may be inaccurate and show poor precision if the intermittency is great. The general trends of the data seem clear, however, and the effect of intermittency was not pursued.

The calculated dispersion coefficient is shown in Figure 6 for water and 50 wppm Polyox. While u' is decreased, T_L is increased with the result that E_D shows a small increase in Polyox solutions.

The effects of Polyox concentration on intensity, scale, and dispersion coefficient are shown in Figure 7. The results described above for 50 wppm Polyox are seen to be generally observed at lower concentrations as well. There is some indication of a maximum effectiveness in altering turbulence parameters at about 20 wppm. This corresponds well with the maximum in drag reduction at 20 wppm observed by Hoyt (1971) and Goren and Norbury (1967). This latter point is shown in Figure 8 constructed principally from Hoyt (1971), but including data of this study.

As mentioned earlier, it is the small-scale structure of turbulence that is believed to be altered by the addition of drag-reducing polymer to the system. Equation (8) allows approximate calculation of the Taylor microscale λ from measurement of T_L:

$$\lambda \approx 5(\nu T_L)^{1/2} \qquad (17)$$

Since both ν and T_L are increased by the presence of Polyox, λ is increased, that is, small scale fluctuations are somewhat suppressed in drag-reducing fluids. This is consistent with the visual observation that in Polyox the dye plume shows large scale intermittency.

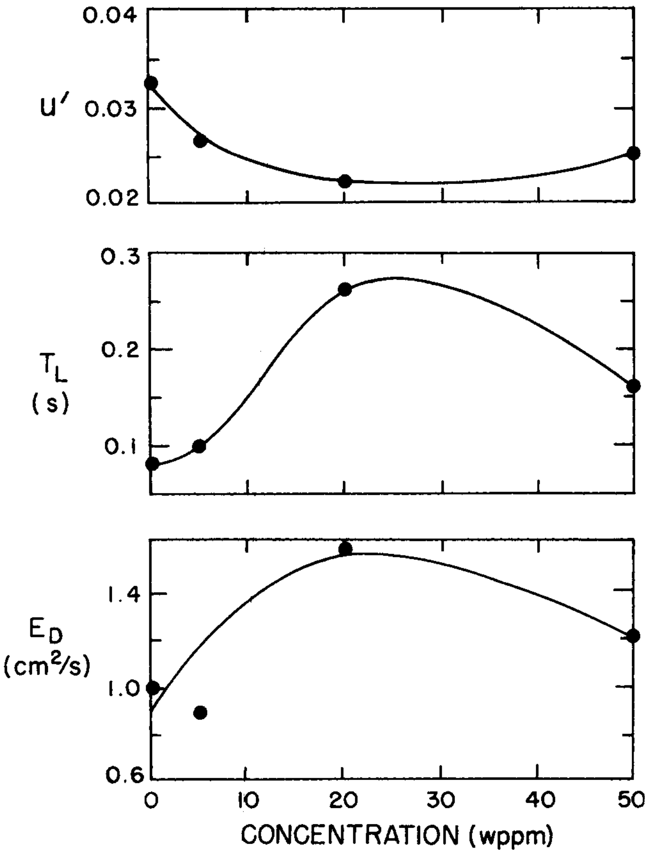

Fig. 7. Dependence of radial turbulence intensity, Lagrangian integral scale, and dispersion coefficient on Polyox concentration.

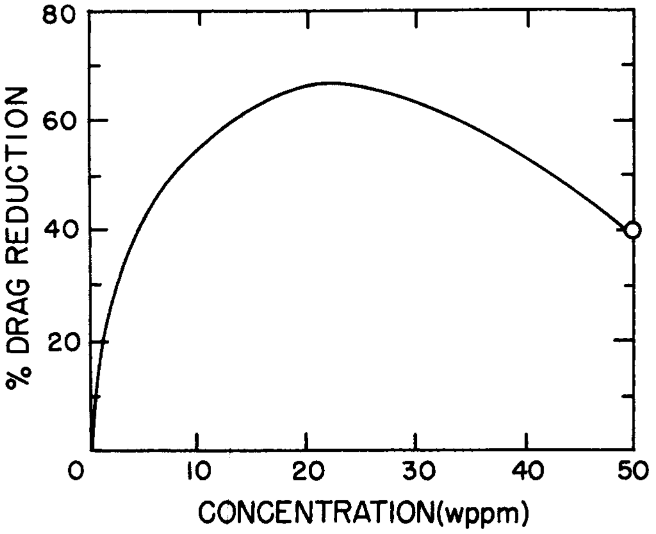

Fig. 8. Dependence of extent of drag reduction on Polyox concentration.

This effect is more dramatic when the dissipation rate is examined. From Equation (7) it can be seen that if $\overline{u^2}$ is reduced and λ is increased, while μ is only slightly increased, the net result is a substantial reduction of energy dissipation in the core of the flow. Table 2 shows dissipation ratios calculated from the data presented herein.

Recall that Equation (7) follows from the assumption of isotropy, which may be followed only in a small region along the pipe axis. Thus the calculated effect on dissipation may be only approximate. Nevertheless, it seems possible to suggest that drag-reducing fluids bring about substantial reductions in turbulent dissipation of energy. The utilization of Equation (8) may also be questioned in the case of drag-reducing fluids, but no alternative is presently available.

It is also of interest to examine the distribution of energy over frequency, as expressed by the energy spectrum function $E_L(n)$. If Equation (11) is used for the correlation coefficient, Equation (9) leads to the result

$$E_L(n) = 4\overline{u^2}\,T_L/(1 + 4\pi^2 T_L^2 n^2) \qquad (18)$$

Figure 9 shows the energy spectra at $N_{Re} = 45{,}000$. The addition of drag-reducing polymer shifts the energy

TABLE 2. EFFECT OF DRAG-REDUCING FLUID ON DISSIPATION RATE

N_{Re}	c(wppm)	$W_{\text{water}}/W_{\text{Polyox}}$
45,000	5	2.0
	20	8.0
	50	3.3
35,000	50	6.7
24,500	50	2.3
17,800	50	1.3

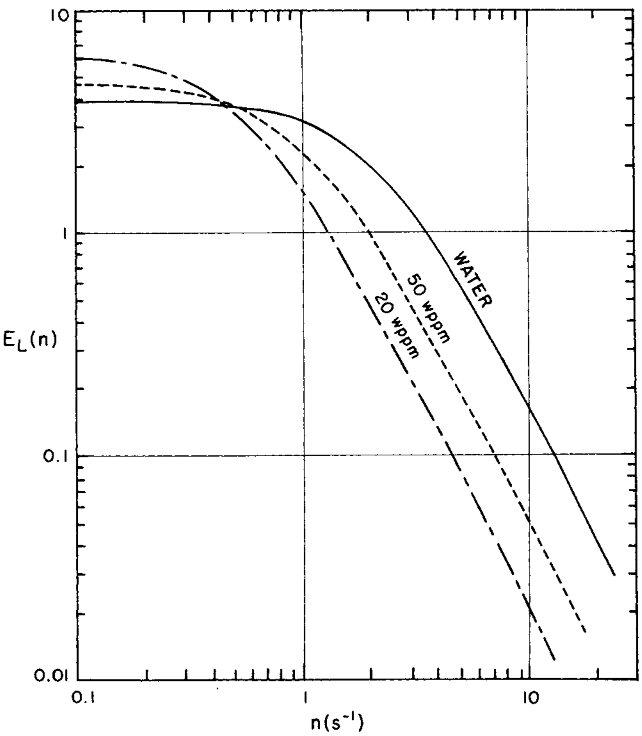

Fig. 9. Effect of Polyox on energy spectra.

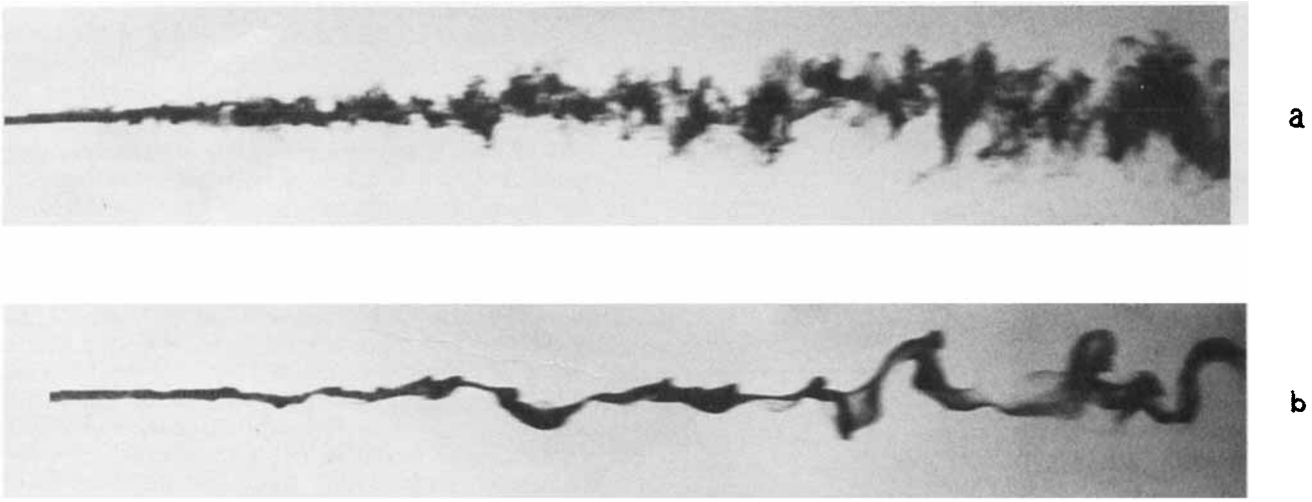

Fig. 10. High speed photographs of dye plumes in water (a) and in 50 wppm Polyox (b), at $N_{Re} = 45{,}000$.

toward low frequency components corresponding to large scale eddies. Again the picture is consistent with other indications presented above.

Visual confirmation of these results was obtained by taking high speed flash photographs of the dye plume. Figure 10 shows a comparison, at $N_{Re} = 45{,}000$, of water and 50 wppm Polyox. Clearly the small-scale structure of the water flow has been suppressed, and dispersion of the plume in the drag-reducing fluid is through large-scale motion.

CONCLUSIONS

1. In drag-reducing fluids, for which standard turbulence measuring equipment such as hot-wire and hot-film probes may be inapplicable, a simple dye-dispersion method is capable of yielding results of acceptable reliability.
2. Under otherwise identical conditions, turbulence in a drag-reducing fluid has a smaller intensity than in the solvent. The distribution of energy is shifted by the presence of the polymer additive to low frequency large-scale fluctuations. As a result the fine structure disappears in the drag-reducing fluid, dispersion occurs principally through large scale motion, and the flow has a more intermittent character.
3. A simple exponential form for the Lagrangian correlation coefficient seems to be superior to other suggested models, a point in agreement with Frankiel (1948), Hinze (1959), and Kalinsky and Pien (1944). This is true despite the fact that the simple exponential model fails to meet all of the theoretical criteria of Frenkiel (1948).
4. Drag-reducing fluids show a significant alteration in the character of turbulence in the core of a pipe flow.

It may not be necessary to invoke a specific, localized, interaction with the pipe wall or its neighborhood in order to explain drag-reducing phenomena. However, such an interaction may in fact be compatible with observed alterations of the core flow. This point will be discussed in a separate and subsequent consideration of mechanisms of drag reduction in viscoelastic fluids.

NOTATION

A = absorbance of dye sample
E_D = dispersion coefficient, cm^2/s
$E_L(n)$ = one-dimensional energy spectrum function
f = friction factor
n = frequency, s^{-1}
N = number of data points, Equation (15)
N_{Re} = Reynolds number
P = number of parameters, Equation (15)
$R_L(\tau)$ = Lagrangian correlation coefficient
T_L = Lagrangian integral time scale, s
t = time, s
u = fluctuating component of radial velocity, cm/s
u' = turbulent intensity
U_c = mean velocity in the core, cm/s
$\overline{u^2}$ = mean squared velocity, cm^2/s^2
W = rate of turbulent dissipation, g/cm-s^3
Y = distance from pipe wall, along diameter, cm
$\overline{Y^2}$ = variance of the concentration distribution, cm^2

Greek Letters

λ = Taylor microscale, cm
μ = viscosity, g/cm-s
ν = kinematic viscosity, cm^2/s
σ^2 = error variance, cm^4
τ = time, s

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Unsteady Mass Transfer in a Long Composite Cylinder with Interfacial Resistances

A solution is given for unsteady state concentration profiles in an infinitely long composite cylinder resulting from a step-change in concentration in a large reservoir surrounding the cylinder. The composite cylinder consists of an inner cylinder with diffusivity D_1 surrounded by a permeable tube with diffusivity D_2 and has interfacial mass transfer resistances at the cylinder-tube and tube-reservoir interfaces. Numerical values are given for the first eight roots and various coefficients of the solution for physical properties typical of tubular (hollow fiber) membranes. These results can be used in the analysis of data from unsteady state mass transfer experiments to determine the permeability of homogeneous tubular membranes. A simple, approximate method for data analysis is suggested for the rapid estimation of the permeability of both homogeneous and asymmetric (skinned) tubular membranes.

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SCOPE

Although permeability measurements are routinely made for flat membranes, no convenient and accurate method is available at present for making these measurements for small tubular (hollow fiber) membranes. In this paper a method based on an unsteady state diffusion experiment is proposed for measuring the permeability of tubular membranes. The objective of this paper is to present the mathematical analysis which is necessary for the interpretation of the experimental results. A detailed description of the experimental method and experimental data will be presented in a subsequent paper.

Small tubular membranes are finding increasing use in mass transfer devices such as the artificial kidney and are now being considered for novel applications such as reaction vessels for enzyme-catalyzed reactions. A convenient and accurate method for measuring the permeability of these membranes will be useful for modeling membrane behavior, evaluating candidate membranes for specific applications, optimizing membrane composition and processing conditions, and maintaining quality control during manufacturing. The analysis presented here can easily be applied to the analogous heat transfer problem